Gödel's Argument for the Ideality of Time

(1) Analytical metaphysics/conceptual analysis

(2) Scientific discoveries

(3) Stir (1) and (2) together and bake lightly ...

Gödel stands at the end of a long line of philosophers who deny the objectivity of change and consider change an illusion: Parmenides, Kant, McT.

Gödel is going to offer an "unequivocal proof" of the ideality of time and change.

General Relativistic Models

$$\langle M, g_{ab}, T_{ab} \rangle$$

M = spacetime manifold

g_{ab} = spacetime metric

T_{ab} = stress-energy tensor

Einstein field equations $F(g_{ab}, T_{ab}) = 0$

1) ***Lightcones***. g_{ab} defines at every $p \in M$ a (local) lightcone. Divides the tangent space into three regions:

lightlike (or null), spacelike, and timelike.

2) ***Time orientability***. The spacetime M, g_{ab} is time orientability iff there is a globally consistent division of the lobes of the lightcones into "past" and "future."

Lemma: If M is simply connected then M, g_{ab} is time orientable.

3) ***Time orientation***. In a time orientable spacetime, choose one of the two possible orientations. (How? That is part of the problem of the direction of time.)

For $p, q \in M$, define $p << q$ (read "p

chronologically precedes q") := $\exists$ a future directed timelike curve from p to q.

Lemma: $<<$ is transitive.

4) ***Time order***. A time oriented spacetime M, g_{ab} possess a consistent time order iff $<<$ is anti-symmetric. This will be the case iff there are no closed future-directed timelike curves (CTCs).

5) ***Global time function***. A global time function is a smooth map

$$t : M \to \mathbf{R} \text{ (real line)}$$

such that $\forall p, q \in M$, $t(p) < t(q)$ if $p << q$.

If such a time function exists, then the level surfaces

$$t = const.$$

are global timeslices (candidates for "Now").

Lemma. M, g_{ab} admits a global time function iff it is stably causal (roughly, there are no CTCs and there is a finite widening of the lightcones that does not result in CTCs).

Gödel's solution to EFE

- T_{ab} describes a pressureless fluid (aka dust); this fluid is everywhere rotating relative to the local compass of inertia.

- $M = \mathbf{R}^4$

- the spacetime is time orientable

- through every point of the spacetime there is a CTC

(Note: These CTCs are not geodesics. So a trip around one of them requires acceleration.)

- there does not exist a single global timeslice (spacelike hypersurface without edges). So the spacetime cannot be partitioned into global "Nows."

Part I

Conceptual analysis + STR ⇒ ideality of time

- Change is possible only through lapse of time.

[An inversion of "time requires change"?]

- Objective lapse of time means an infinity of layers of "now" which come into existence successively.

[Growing block model?]

- STR teaches us that reality cannot be split into such layers in an objective manner.

- But can lapse of time be relative (say, to an observer) but still objective? No–see fn. 5.

Part II

GTR seems to undercut Part I

- The cosmological models of GTR known at time (circa 1940s) had the property that the smoothed out matter distribution is non-rotating.

- Frobenius' theorem: A congruence of timelike worldlines is non-rotating iff it is orthogonal (in the sense of g_{ab}) to a family of spacelike hypersurfaces.

- Natural to take the family of hypersurfaces orthogonal to the smoothed out matter flow as "Nows" that serve as the basis for an objective lapse of time.

Part III

Gödel solution to the rescue

- In the Gödel solution matter is everywhere rotating. So the procedure of Part II does not apply.

- Stronger (??) argument (not offered by Gödel): In the Gödel solution there are no global time slices and, thus, no basis for an objective or relativized lapse of time.

Part IV

Filling the gap

Granted that there is a physically possible universe in which there is no objective lapse of time.

How does it follow that there is no objective lapse of time in our universe, especially since in our universe matter is not everywhere rotating and there does exist a global time function whose level surfaces are orthogonal to the worldlines of matter (at least if the large scale features of our universe conform to the standard FRW model).

Gödel's response: Whether or not there is an objective lapse of time should not depend on contingencies like the distribution of matter.

Side issue: Is the Gödel universe really a physically possible universe? Paradoxes of time travel.

Gödel's response: Recall that CTCs involve acceleration. Compute the fuel requirements needed to complete a trip around a CTC.

Total acceleration along a timelike curve γ:

$$TA(\gamma) := \int_{\gamma} a d\tau$$

where a is the magnitide of of the (four-) acceleration and τ is proper time along γ.

$$\frac{m_p}{m_p + m_f} \leq \exp(-TA(\gamma))$$

where m_p is the mass of the payload and m_f is the mass of the fuel.

Conjecture (Malament): The greatest lower bound on $TA(\gamma)$ as γ ranges over the CTCs in the Gödel universe is $2\pi(9 + 6\sqrt{3}) \approx 27.67$.